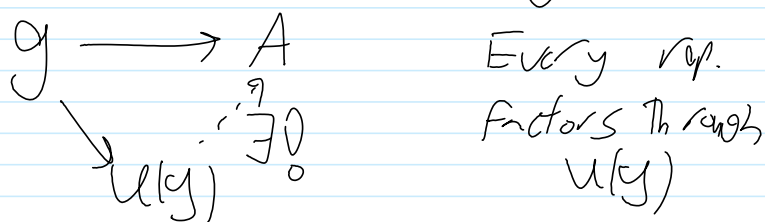


HW 6 returned, lost compensation for bad formulation of Q2.
 HW 7 due. HW 8 on web.

Today's goals: univ. env. algebras, The bi-algebra structure

Given \mathfrak{g} , $\mathcal{W}_{\mathfrak{g}}: \mathcal{A}(\uparrow) \rightarrow \mathcal{U}(\mathfrak{g})$:

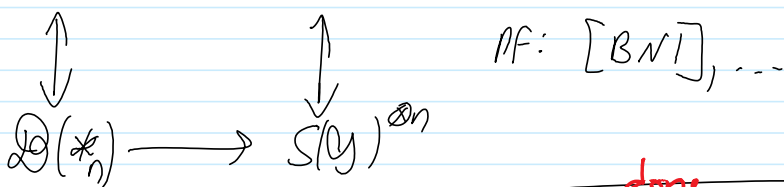
1. $\mathcal{U}(\mathfrak{g})$: The "universal enveloping algebra"



2. \mathcal{A} is a "universal universal enveloping algebra"

3. $\mathcal{W}_{\mathfrak{g}}: \mathcal{A}(\uparrow_n) \rightarrow \mathcal{U}(\mathfrak{g})^{\otimes n}$

4. PBW: $\mathcal{A}(\uparrow_n) \rightarrow \mathcal{U}(\mathfrak{g})^{\otimes n}$



\mathcal{A} is an algebra and so is \mathcal{U} . Is \mathcal{A} a bi-algebra?

1. Define algebra $(A, m, \epsilon, \text{diagrams})$

2. Define co-algebra $(C, \Delta, \eta, \text{diagrams})$

3. Define bi-algebra.

4. \mathcal{A} is a bi-algebra

$$\exists \Delta: \mathcal{A} \rightarrow \mathcal{A} \otimes \mathcal{A} \text{ s.t. } \forall v_1, v_2 \in \mathcal{U}$$

$$W_{v_1 v_2} = m_{\otimes} \circ (W_{v_1} \otimes W_{v_2}) \circ \square_{\mathcal{A}}$$

5. Milnor-Moore & primitives.

6. Warning: Given \mathfrak{g}

$$(\mathcal{A}, m, \square) \leftrightarrow (U(\mathfrak{g}), m, \Delta)$$

Questions 1. What is Δ in \mathcal{A} language?

2. What is \square in $U(\mathfrak{g})$ language?